

A Two-Field Quintessence Model

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We study the dynamics of a quintessence model based on two interacting scalar fields. The model can account for the (recent) accelerated expansion of the Universe suggested by astronomical observations. Acceleration can be permanent or temporary and, for both scenarios, it is possible to obtain suitable values for the cosmological parameters while satisfying the nucleosynthesis constraint on the quintessence energy density. We argue that the model dynamics can be made consistent with a stable zero-energy relaxing supersymmetric vacuum.

Recent observations of type Ia supernovae, together with Cosmic Microwave Background (CMB) and cluster mass distribution data [1–3] indicate that the Universe is flat, in agreement with the inflationary prediction, accelerating and that the energy density of (baryonic plus dark) matter is smaller than the critical density. Thus, observations suggest that the dynamics of the Universe at present is dominated by a negative pressure component, the main candidates being a cosmological constant and a slowly-varying vacuum energy, usually referred to as dark energy or quintessence [4,5] (an evolving vacuum energy was discussed somewhat earlier, e.g. [6]). The main difference between the cosmological constant and quintessence scenarios is that, for quintessence, the equation of state parameter, $w_Q \equiv p/\rho$, varies with time and approaches a present value $w_Q < -0.6$, whilst for the cosmological constant, it remains fixed at $w_\Lambda = -1$.

Several quintessence models have been put forward, most of them based on a scalar field which was subdominant in the early Universe and, more recently, has started to dominate the energy density of non-relativistic matter. Theoretical suggestions include a scalar field endowed with exponential [7–10] or inverse power law potentials [11], the string theory dilaton in the context of gaugino condensation [12], an axion field with an almost massless quark [13], scalar-tensor theories of gravity [14], or one of the fields arising from the compactification process in the multidimensional Einstein-Yang-Mills system [15]. Some of these models address the “cosmic coincidence” problem *i.e.* the question of explaining why the vacuum energy or scalar field dominates the Universe only recently. In tracker models, the tracker field rolls down a potential according to an attractor-like solution to the equations of motion, causing the energy density of the quintessence field to track the equation of state of the background energy component independently of ini-

tial conditions [11]. However, in these models, the overall scale of the potential has to be fine-tuned in order for the quintessence energy to overtake the matter density at present.

In k -essence models [16], as a result of the dynamics, tracking of the background energy density can only occur in the radiation epoch; at the onset of matter-domination, the k -essence field energy density drops sharply, increasing again and overtaking the matter energy density at roughly the current epoch. At least in the original proposal, these features require the introduction of a non-linear kinetic energy density functional of the scalar field and adjusting it to obtain the desired attractor behaviour.

A common feature of the proposals mentioned above is that the asymptotic accelerating behaviour of the Universe is driven by the dynamics of a single field. In this work, we shall consider instead a two-field model. Two-field quintessence models were previously considered, in an attempt to explain how to obtain a small but non-vanishing cosmological constant [17] and in the context of SUSY QCD [18]. Actually, there are several motivations for studying potentials with coupled scalar fields. In fact, if one envisages to extract a potential suitable for describing the Universe dynamics from fundamental theories, it is most likely that an ensemble of scalar fields (moduli, axions, chiral superfields, etc) will emerge, for instance, from the compactification process or from the localization of fields in the brane in multi-brane models or from mechanisms responsible for the cancellation of the cosmological constant (see e.g. [19] and references therein). Furthermore, coupled scalar fields are invoked for various desirable features they exhibit, as in the so-called hybrid inflationary models [20,21] and in reheating models in the presence [22] or absence [23] of parametric resonance. Finally, it has been recently pointed out that

an eternally accelerating Universe poses a challenge for string theory, at least in its present formulation, since asymptotic states are inconsistent with spacetimes that exhibit event horizons [24]. Moreover, it is argued that theories with a stable supersymmetric vacuum cannot relax into a zero-energy ground state if the accelerating dynamics is guided by a single scalar field [24]. In this paper, we present a two-field model whose solutions allow for, at least, a partial fixing of these inconsistencies.

Another interesting feature of our model is that it presents two types of solutions, namely one in which the Universe accelerates forever and one in which it is possible for the Universe to exit from a period of accelerated expansion and resume decelerated expansion. The latter type of solution is compatible with the conceptual framework underlying string theory.

It is believed that scalar fields with potentials of the type

$$V(\phi, \psi) = e^{-\lambda\phi} P(\phi, \psi) , \quad (1)$$

where $P(\phi, \psi)$ contains polynomial as well as interacting terms in ϕ and ψ , arise in the low-energy limit of fundamental particle physics theories such as string/M-theory [25], $N = 2$ Supergravity coupled with matter in higher dimensions [26] and phenomenological brane-world constructions [19,27]. The overall negative exponential term in ϕ signals that this is a moduli type field which has acquired an interacting potential with the ψ field. A simple possibility is

$$P(\phi, \psi) = A + (\phi - \phi_0)^2 + B (\psi - \psi_0)^2 + C \phi(\psi - \psi_0)^2 + D \psi(\phi - \phi_0)^2 , \quad (2)$$

in units where $M \equiv (8\pi G)^{-1/2} = \hbar = c = 1$. Notice that this potential, for $B = C = D = 0$, in which case only the ϕ field is present, coincides with the one proposed in Ref. [9], hereby referred to as the AS model. We have considered just tree interacting terms as they already capture the main aspects of the coupled dynamics we are interested in. As we will show, an important property of model is that it can lead to an asymptotic dynamics where either ψ or both fields do not necessarily settle in their minima at present, which is the key to evade some of the conclusions of Refs. [24], concerning the stability of the supersymmetric vacua.

We consider a spatially-flat Friedmann-Robertson-Walker (FRW) Universe containing a perfect fluid with barotropic equation of state $p_\gamma = (\gamma - 1)\rho_\gamma$, where γ is a constant, $0 \leq \gamma \leq 2$ (for radiation $\gamma = 4/3$ and for dust $\gamma = 1$) and two coupled scalar fields with potential given by Eq. (1). The evolution equations for a spatially-flat FRW model with Hubble parameter $H \equiv \dot{a}/a$ are

$$\dot{H} = -\frac{1}{2} \left(\rho_\gamma + p_\gamma + \dot{\phi}^2 + \dot{\psi}^2 \right) , \quad (3)$$

$$\dot{\rho}_\gamma = -3H(\rho_\gamma + p_\gamma) , \quad (4)$$

$$\ddot{\phi} = -3H\dot{\phi} - \partial_\phi V , \quad (5)$$

$$\ddot{\psi} = -3H\dot{\psi} - \partial_\psi V , \quad (6)$$

where $\partial_{\phi(\psi)} V \equiv \frac{\partial V}{\partial \phi(\psi)}$, subject to the Friedmann constraint

$$H^2 = \frac{1}{3} \left(\rho_\gamma + \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\psi}^2 + V \right) , \quad (7)$$

The total energy density of the homogeneous scalar fields is given by $\rho_Q = \dot{\phi}^2/2 + \dot{\psi}^2/2 + V(\phi, \psi)$.

A necessary and sufficient condition for the Universe to accelerate is that the deceleration parameter, q , given by

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = \frac{1}{2}(1 + 3w_Q\Omega_Q + \Omega_r) , \quad (8)$$

where Ω_r is the fractional radiation energy density, is negative.

We now study how the solutions of the system above depend on the parameters of the potential and initial conditions. We integrate from $N=-30$, corresponding to the Planck epoch; nucleosynthesis occurs around $N=-10$, radiation to matter transition around $N=-4$ and $N=0$ today.

There are essentially two realistic types of behaviour, illustrated in Figures 1 (Model I) and 2 (Model II). Model I ($\lambda = 9.5, A = 0.02, \phi_0 = 29, \psi_0 = 15, B = 0.002, C = 6 \times 10^{-4}, D = 4.5$) corresponds to the case where vacuum domination, which occurs when $\Omega_{Q0} > 1/2$, is permanent and Model II ($\lambda = 9.5, A = 0.1, \phi_0 = 29, \psi_0 = 20, B = 0.001, C = 8 \times 10^{-5}, D = 2.8$) to the case where vacuum domination is transient. Permanent and transient vacuum domination have also been found in the AS model, in Refs. [9] and [10], respectively. Of course, there remains the (non-realistic) case where accelerated expansion never occurs.

In both models the equation of state has reached $w_Q \simeq -1$ for the present time, as favored by the available data [28] (and making it hard to distinguish from a cosmological constant) but, whereas in Model I w_Q remains negative, in Model II it is in the process of increasing today towards positive values, then oscillates slightly until it reaches its asymptotic value. Similarly, in both models, the deceleration parameter is negative today but, whereas for Model I q remains negative, in Model II it oscillates and becomes positive before it reaches its asymptotic value.

Permanent vacuum domination takes place when at least the ϕ field ends up settling at the minimum of the potential, thus corresponding to a cosmological constant; in Model I, both fields settle at the minimum of the potential, see Figure 3. Transient vacuum domination occurs either when the potential has no local minimum or ϕ arrives at the local minimum with enough kinetic energy to roll over the barrier and resume descending the potential. Notice that the evolution of ψ is slight compared with ϕ , especially for Model II.

Both models satisfy present bounds on relevant cosmological observables. The tightest bound comes from nucleosynthesis, $\Omega_Q(N \sim -10) < 0.044$, requiring $\lambda > 9$

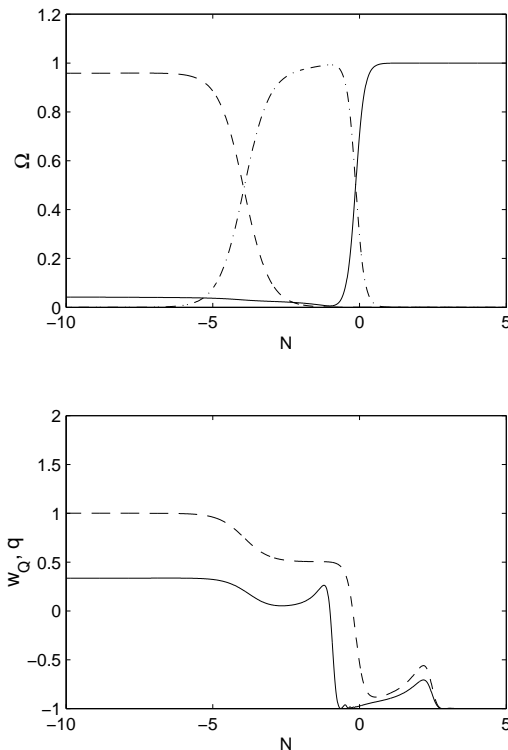


FIG. 1. The upper panel shows the evolution of Ω_Q (solid), Ω_r (dashed) and Ω_m (dot-dashed) for Model I, corresponding to permanent vacuum domination. The lower panel shows the evolution of w_Q (solid) and q (dashed).

[29]. The bound arising from the most recent CMB data, $\Omega_Q < 0.39$ at last scattering, is less stringent than the nucleosynthesis bound. Other bounds we take into account are: $\Omega_m = 0.3 \pm 0.1$, $w_Q < -0.6$, $\Omega_Q = 0.65 \pm 0.05$ [30] and $h = 0.65 \pm 0.05$ [3] today. Indeed, Model I has $h = 0.6$, $\Omega_Q = 0.7$, $\Omega_m = 0.3$, $w_Q = -1$ and $q = -0.5$ today, $\Omega_Q = 0.042$ at nucleosynthesis. Similar values are found for Model II, namely $h = 0.6$, $\Omega_Q = 0.7$, $\Omega_m = 0.3$, $w_Q = -0.9$ and $q = -0.4$ today, $\Omega_Q = 0.042$ at nucleosynthesis.

Our models seem to be more sensitive to changes in the initial conditions than models with just one scalar field and, in particular, the AS model (this is to be expected since there is more freedom e.g. in the way kinetic energy is shared between the two fields) but no fine tuning of the initial conditions is needed. Indeed, fixing e.g. $x = z$, corresponding to equipartition of kinetic energy between ϕ and ψ , we have studied the behaviour of ρ_Q and seen that, for a wide range of the remaining initial conditions, after some initial transient, each solution scales with the dominant matter component until ρ_Q begins to dominate.

We have studied the nature of our solutions for a rather broad range of parameters of the potential. We have found that it is possible to obtain permanent or transient vacuum domination, satisfying present bounds on observable cosmological parameters, for various combi-

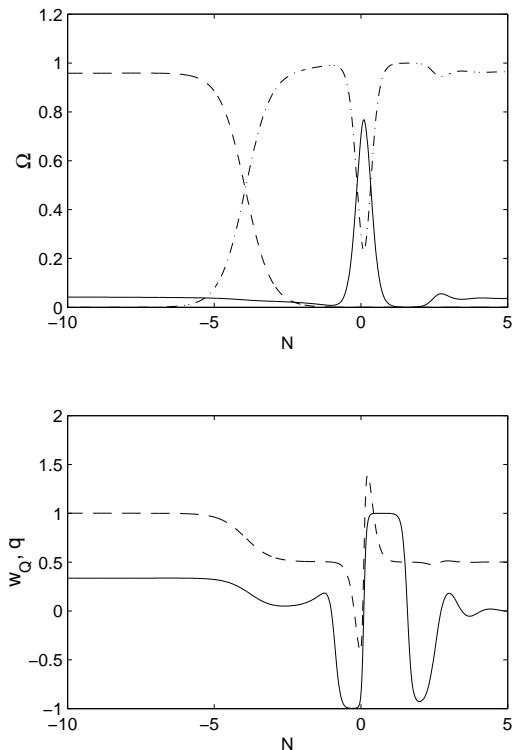


FIG. 2. The upper panel shows the evolution of Ω_Q (solid), Ω_r (dashed) and Ω_m (dot-dashed) for Model II, corresponding to temporary vacuum domination. The lower panel shows the evolution of w_Q (solid) and q (dashed).

nations of the potential parameters.

A relevant issue of our proposal is that it allows evading the conclusions of Refs. [24], in what concerns the stability of a supersymmetric potential. The main argument presented in [24] relies on the fact that, in a supersymmetric theory, one expects that the asymptotic behaviour of the superpotential is given by $W(\phi) = W_0 e^{-\alpha\phi/2}$, which, in order to ensure the positivity of the 4-dimensional potential $V(\phi) = 8|\partial_\phi W|^2 - 12|W|^2$ implies that $|\alpha| > \sqrt{6}$. However, this value is inconsistent with the requirement of an accelerated Universe at present $|\alpha| = \sqrt{3(1+\omega_{Q0})}/2 < 1.5$ [4] as data suggest that $\omega_{Q0} < -0.6$ [30]. The situation is different in the presence of fields that do not reach their minima asymptotically, as in Model II. Indeed, in this case, the asymptotic behaviour of the superpotential would be better described by the function $W(\phi) = W_0 e^{-\alpha\phi/2} F(\phi, \psi)$, where $F(\phi, \psi)$ is a polynomial in the fields ϕ and ψ . The positivity condition now reads: $\alpha^2 - 6 + 4[(\partial_\phi F)^2 + (\partial_\psi F)^2]/F^2 > 0$. One can then easily see that, by a suitable choice of the polynomial $F(\phi, \psi)$, the positivity condition can be reconciled with the requirement of successful quintessence. Furthermore, since in Model II acceleration is transient and occurs only at present, this model is consistent with the underlying framework of string theory as it does not present cosmological horizons that are associated with eternally accelerating universes.

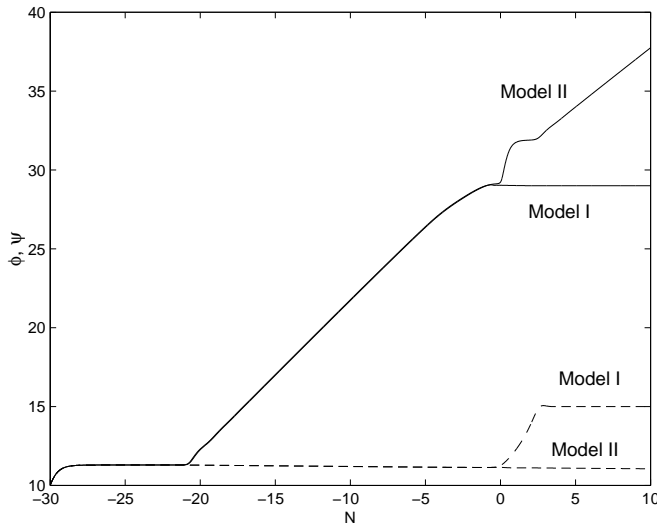


FIG. 3. Evolution of the quintessence fields, ϕ (solid) and ψ (dashed), for Models I and II.

Solutions corresponding to transient acceleration have not been found in previous two-field quintessence models. In the model of Ref. [18], where the fields invoked are the vacuum expectation values of SUSY QCD chiral superfields, quintessence energy density grows with respect to matter as $\rho_Q/\rho_m \sim a^{3(1+r)/2}$, where r is the ratio between the number of flavours and the number of colours. Similarly, in the model of Ref. [17], where the potential $V(\sigma, \Phi) = e^{-4\zeta\sigma}(\Lambda + \frac{1}{2}m^2\Phi^2[1 + \gamma \sin(k\sigma)])$ is proposed, quintessence energy density dominates matter energy density asymptotically.

We conclude that the late time dynamics arising from our two-field potential is consistent with the observations as well as the theoretical requirements of stability of the supersymmetric ground state and the asymptotic behaviour of string theory states, provided the observed accelerated expansion of the Universe is transient and decelerated expansion is soon resumed, as in Model II. This solution has been recently proposed to solve the contradiction between accelerated expansion and string theory [31], on general grounds; in this work, we have presented a concrete example of a two-field model that exhibits this desirable feature.

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